13 Jul 2000

Large Eddies and Vortex Streets Behind Moving Jets in a Stratified Fluid

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1. Abstract

The results of experiments with jet-like flows, induced by a horizontally moving momentum source in a density-stratified fluid are presented. The jet acts either impulsively or continuously. In the latter case both the counter- and co-flowing motions are considered. It is shown that large eddies and vortex streets, similar to those observed behind bluff bodies, are formed in the flow, where the effect of solid boundary is negligible. The conditions under which large eddies and vortex streets appear in the flow are determined. Possible applications include stratified wakes behind maneuvering self-propelled bodies.

2. Introduction

When a self-propelled body makes a maneuver (e.g., accelerates), the drag force is not identically equal to the thrust during the maneuver, which lasts for a time interval Δt. Experiments in a stratified fluid show (Voropayev et al. 1999) that in this case the formation of unusually large eddies is possible. The late flow consists either of a large dipolar eddy or a system of smaller eddies organized in a vortex street (similar to that behind a sphere in a stratified fluid, see, e.g., Pao & Kao, 1977, Lin et al., 1992; Spedding, 1997). The direction of the vortex street depends on the correlation between the drag and the thrust. In a still fluid the lifetime of the formed structures is several orders of magnitude larger than the time interval Δt, but the background shear may reduce this time (Voropayev et al. 2000). This pronounced feature of the stratified wakes was explained qualitatively as follows (Voropayev et al., 1999): during the time interval Δt the self-propelled body applies a force to the fluid, so that the wake behind it acquires a momentum, which is an integral of motion. The organization of eddies in the late flow is governed by this quantity and does not depend strongly on the particular distribution of vorticity at the surface of the body. It is also plausible that the amount of momentum determines the formation and evolution of the late wake eddies and their characteristics. In order to verify this idea, we conducted a series of experiments with a moving source, which produces a controllable momentum while the effect of a solid boundary is negligible. For this purposes a moving momentum source (jet), for which solid body drag is negligibly small compare to the momentum flux, was used in the experiments. Two basic configurations are considered, namely, continuously and impulsively acting jets. In the former case counter- and co-flows were reproduced.

3. Experimental set-up

The experiments were conducted in a long rectangular tank 400x30x40 cm filled with a stratified by salt water (buoyancy frequency N). Vertical profiles of conductivity (density) were measured using standard four-electrode micro-scale conductivity probe. The momentum flow with controllable intensity J was generated using a thin vertical L-shaped tube (diameter $d = 0.13$ cm), which was driven with the velocity U along the tank by a computer controlled traverse mechanism. By measuring the amount of dyed fluid (volume flux q), which is injected either impulsively or continuously through the nozzle into the surrounding fluid, the (kinematic) momentum flux J, transported by the source to the fluid, can be accurately estimated as $J = q(q-Us)/s$ ($s = \pi d^2/4$). The vertical location of the source was near the midplane of the tank. The flow was visualized by the thymol-blue pH-indicator (Voropayev et al., 1991) and videotaped from above using super-VH video camera. The quantitative data were obtained from video records and the images of the typical flow patterns given below were obtained by digitizing selected frames from video.

4. Results

Impulsive jet. For impulsively acting jet the fluid was injected during relatively short time interval Δt in the direction opposite to the source motion. The net flow momentum is equal to $I = \Delta t\, J$. A typical example of the flow is shown in Fig. 1. The source moves from left to right and acts in the opposite direction. At early times $t \leq \Delta t$ (Fig. 1a), a three-dimensional turbulent conical cloud, very similar to that in a homogeneous fluid, is formed,. With time elapse (Fig. 1b), the vertical component of velocity decays due to the stratification and the flow takes typical pancake shape (Fig. 1c). The resulting flow pattern strongly depends on the values of the system parameters. In some cases large dipolar eddy can be observed in the flow (Fig. 1d), in other cases only a system of small eddies remains. If the large eddy is formed, and this occurs through the bifurcation and merging of the vorticity cloud branches (Fig. 1c), its location coincides with the front region of the initial jet flow. As soon as it is formed, it moves forward, entraining the surrounding fluid and increasing its horizontal size (Fig. 1d). The active lifetime of the eddy may be as long as many tens of minutes, which is much greater than the duration of forcing. Eventually, at large times it decays because of the viscous diffusion of the vorticity in the vertical direction. The dipolar pattern, visualized by a dye, may remain visible during hours, and this fossil structure moves slowly in a large-scale potential flow, which was induced by the pressure forces in the whole body of fluid at the moment of the source action.

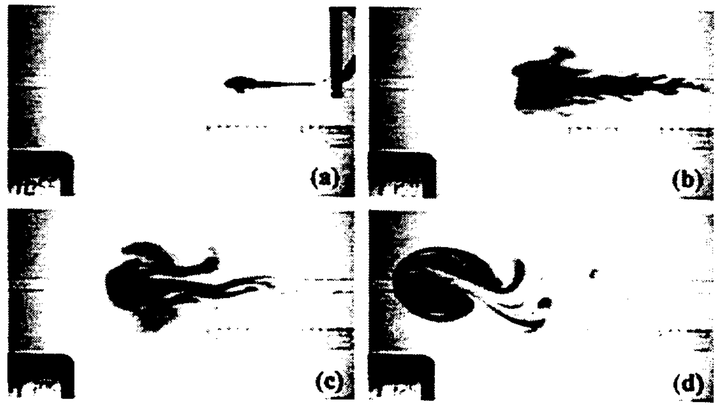

Figure 1. Formation of large dipolar eddy in a flow induced by a moving impulsive jet in a stratified fluid. Top view. Experimental parameters: $N = 2.1\ s^{-1}$, $U = 2.0\ cms^{-1}$, $Re = 470$, $\Delta t = 5$ s, $A = 4.6$. The photographs were taken at (a) $t = 0$, (b) 5, (c) 10, (d) 30 s. Scale is given in cm and total length of a ruler is 15 cm.

To estimate the condition of the large eddy formation lets consider simple physical model. The total impulse I, imparted by the momentum source to the fluid, is an integral of motion. Half of it goes to the formation of the vorticity field, the other half is spent on the redistribution of the pressure in the system, so that the dipolar potential flow is created in the whole body of fluid (Cantwell, 1986). The total vorticity of the system remains zero, but the "impulse" of the vorticity distribution (flow momentum, Lamb, 1932) conserves. For a still source (U=0) the fluid behind the front region of the flow progresses approximately twice as fast as the front region itself (Stern & Voropayev, 1984). Therefore, the vorticity generated by the source is advected into the frontal region where it accumulates, leading to the formation of the large dipolar eddy. The formation time t_0 is approximately twice as large as the duration of the source action Δt and does not depend on the intensity of the forcing (Voropayev et al., 1991). For a moving source the motion of the source significantly influences the formation time. Assuming that the stratification does not change significantly the velocity distribution in the co-flowing jet (Rajaratnam, 1976) and that $t_0 > N^{-1}$, the following estimate may be derived (Voropayev et al, 1999): $t_0 = C_1 \Delta t (e^{A/3} - 1)/A$, where C_1 is a constant and $A = U^2 \Delta t^{3/2} / I^{1/2}$ is the dimensionless governing parameter. From here it follows, that only for some values of the external parameters the formation time t_0 is comparable with Δt, and the eddy can be formed. Using the results of 30 experiments conducted at different values of U and I (J and Δt were also varied), the critical value of A was determined as $A_0 = 25$. Thus, the conditions of large eddy formation in a flow, induced by a moving impulsive momentum source in a stratified fluid (Fig. 1), can be estimated as $A < A_0$.

Continuous jet. Consider now the momentum source, which acts continuously. Consider first the counter-flowing motion. A typical example is shown on Fig. 2. The source moves from left to right and acts in the same direction. As can be seen, a liquid blob (filled with dyed fluid) is attached ahead of the source (Fig. 2a) and it keeps moving with the source velocity. The flow in the blob remains three-dimensional and doesn't feel the stratification. Regular vortex street, similar to those observed in the wake of bluff bodies, is formed behind it (Fig. 2b). At this stage ($t > N^{-1}$) collapse has already occurred and the motion behind the source is mostly horizontal. The resulting vortex street (with typical spacing λ) slowly drifts in the direction of the source and its size λ slowly increases (Fig. 2c,d). The vortex street remains visible for many tens of minutes, until it finally disappears because of the viscous diffusion.

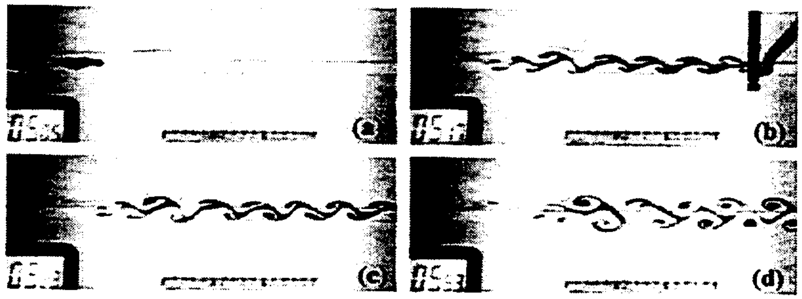

Figure 2. Formation of vortex street in a flow induced by a moving continuously acting jet (counter-flow) in a stratified fluid. Top view. Experimental parameters: $N = 2.1\ s^{-1}$, $U = 2.0\ cms^{-1}$, Re = 75. The photographs were taken at (a) t = 0, (b) 12, (c) 18, (d) 30 s. Scale is given in cm and total length of a ruler is 15 cm.

The jet flow can be characterized by the Reynolds Re and Froude Fr numbers

$$Re = J^{1/2}/\nu, \qquad Fr = U^2/NJ^{1/2} \tag{1}$$

(ν is the fluid viscosity) and the dimensionless time Nt. From the first glance, such parameters differ significantly from parameters $Re^* = UD/\nu$ and $Fr^* = U/ND$, which are traditionally used for a solid body of the size D. But this is not so, and they are closely related to each other. For the problem of the localized momentum source one has: $A_i = F_i\,(J, U, \nu, N, t)$, where A_i is the flow characteristic and F_i is a function. Standard dimensional analysis shows that five variables can be reduced to three independent dimensionless parameters, which are chosen as Re, Fr and Nt. For the problem of solid body the momentum flux (drag force) produced by a moving body can be estimated as $J = (\pi/8)\ CU^2D^2$ (C is the drag coefficient). This gives: $Re^* = (8/C\pi)^{1/2}Re$ and $Fr^* = (C\pi/8)^{1/2}\ Fr$. Thus, the problem of a moving momentum source, in general, is closely related to the problem of a moving body and both these problems are characterized by three similar dimensionless parameters.

Depending on the value of the Reynolds number, two different kinds of vortex streets were observed behind the source. One is a laminar vortex street (Re<100-150), which appears in the immediate wake behind the blob (Fig. 2). It seems, that it is caused by the shear instability of the flow and subsequent meandering. The other is a turbulent vortex street (Re>150), which develops in the late wake far downstream from the highly disordered flow near the source. Some of the vortex street characteristics are given below (Fig. 6).

The length L of the attached blob can be estimated equating its propagating velocity U to the mean velocity $u = C_2 J^{1/2}/x$ (Schlichting, 1979) at the axis of a turbulent jet near the stagnation point at $x = L_0$. This gives

$$L_0 = C_2 J^{1/2}/U. \tag{2}$$

Comparison of the measured L and estimated L_0 values for a range of Re is given in a graph in Fig. 3. Dashed line shows the estimate (2) with $C_2 = 5$, which is only insignificantly differs from Schlichting's estimate $C_2 = 6$.

Note, that at small Re the problem may be solved analytically using Ozeen approximation. The stream function Ψ is given by

$$\Psi = -\frac{1}{2}Ur^2 \sin^2\theta + \frac{9\pi U a^2}{\sqrt{Re}}(1-\cos\theta)\left(1-\exp\left(-\frac{\sqrt{Re}}{12\pi}\frac{r(1+\cos\theta)}{a}\right)\right) \tag{3}$$

and it describes a steady flow induced by a moving momentum source in a viscous fluid ($a = J/6\pi\nu U$, the flow is axis-symmetric and polar coordinates r, θ are used). The streamline $\Psi = 0$ gives a closed contour with attached liquid blob.

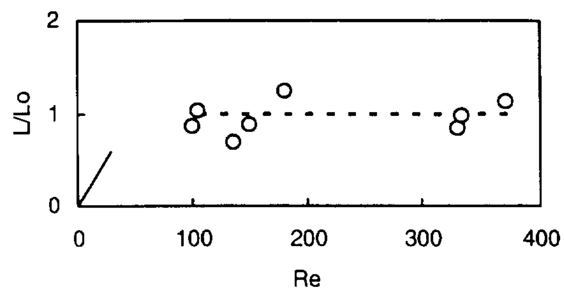

Figure 3. The ratio of the measured L to estimated L_0 (by (2) with $C_2 = 5$) values of the attached blob size as a function of the Reynolds number Re. Solid line shows the tendency for small Re in accordance with (3).

In the case of co-flowing motion the momentum source moves from left to right and acts in the opposite direction (Fig. 4). Initially (Fig. 4a), the flow resembles that produced by a jet in a homogeneous fluid with small-scale disturbances near its boundary. In a short time ($t > N^{-1}$) the flow becomes influenced by the stratification and starts developing into a quasi-two-dimensional, and later (Fig. 4b), a large-scale coherent vortex street is formed. The direction of this street coincides with the direction of momentum applied and it is opposite to that behind a bluff body moving in the same direction as the momentum source. One of the plausible mechanisms of the vortex street formation might be related to the shear instability. Indeed, a strong mean motion, which is characterized by the Reynolds number, persists in the flow. It is worth to mention that the size d of the momentum source is negligibly small (at least two orders of magnitude) compare to the typical street spacing λ.

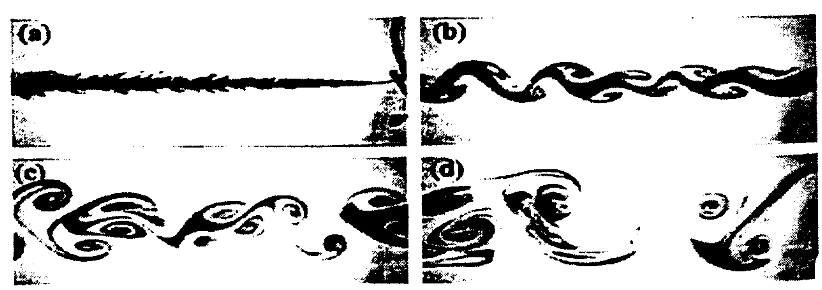

Figure 4. Formation of vortex street in a flow induced by a moving continuously acting momentum source (co-flow) in a stratified fluid. Top view. Experimental parameters: $N = 2.2\ s^{-1}$, $U = 7.9\ cms^{-1}$, Re = 740. The photographs were taken at (a) t = 0, (b) 22, (c) 48, (d) 125 s. Horizontal size of each frame is 100 cm.

Control experiment shows that the moving momentum source itself (L-shaped tube) doesn't produce any significant disturbance in the flow when q = 0. Therefore, the characteristic wavelength λ of the vortex street, which is equal to the distance between two adjacent vortices of the same sign, is determined mostly by the instability of the flow and not by the solid part of the momentum source. As time progresses the coherent vortex street becomes unstable and undergoes a transformation during which dipolar eddies start merging (Fig. 4c). Finally, the flow takes the form of a new vortex street with a doubled wavelength (Fig. 4d). Because of some difference in the strength of initial eddies, the merging doesn't occur regularly along the whole vortex street. As a result, the secondary street (Fig. 4d) is not so ordered as the primary one (Fig. 4b). The resulting eddies last for many tens of minutes until they vanish due to the viscous diffusion. In experiments with larger Re the secondary street demonstrate a strong tendency to a secondary global instability with subsequent merging of eddies and the formation of tertiary street, but to make definite conclusions much larger tank is needed.

Typical time dependencies of the vortex street wavelength λ and its width H (distance between two chains of vortices of opposite sign), as measured in one of the experiments, are shown on a graph in Fig. 5. The data are presented in dimensionless form and L_0 and N^{-1} are used as typical length and time scales. The primary vortex street in the test section was formed in a time interval Nt = 10-15 after the passage of the momentum source (first points on the graph). After the initial stage, the wavelength of the street remains almost constant for a long time while the width of the street H keeps growing slowly. The first stage is followed by the merging of the neighboring eddies. This bifurcation lasts for a period of time Nt = 40-50 and finishes by the appearance of a secondary vortex street with approximately doubled wavelength.

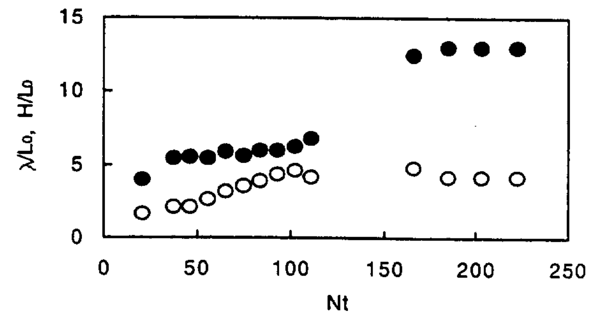

Figure 5. Dimensionless wavelength λ/L_0 (solid circles) and width H/L_0 (circles) of the vortex street as functions of the dimensionless time Nt. Experimental parameters: $N = 1.8\ s^{-1}$, $U = 6.5\ cms^{-1}$, Re = 760.

Similar results were obtained in other experiments with different values of Re and approximately the same values of Fr (= 3-5) and the mean values of λ for the primary vortex street were determined. Using these data, the Strouhal number St = L_0/λ was calculated and its values for different Re and Fr are shown in a graph in Fig. 6. The data are given for both co-flowing (circles) and counter-flowing (solid circles) configurations. As can be seen, the absolute values of St do not differ significantly for both configurations conducted with approximately the same values of Re and Fr. This gives the assurance that the same scaling length L_0 can be used for both configurations. Note, that the data in Fig. 6 demonstrate the general trend for St to decrease with the increase of Re.

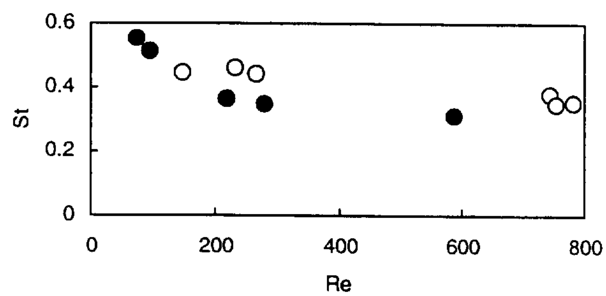

Figure 6. Inverse dimensionless wavelength St = L_0/λ of the vortex street as a function of Reynolds Re and Froud Fr numbers. Data for co-flow (circles) and counter-flow (solid circles) configurations are shown.

5. Conclusions

Results of experiments show that the momentum source (jet) moving in a stratified fluid generates late wake eddies which are similar to those generated by a solid body. The effect of solid parts (in the momentum source) on the flow is negligibly small compare to the effect of the source itself, and this indicates that the mechanism of large eddy formation is not strongly related to the shedding of vorticity from the solid boundary. For an impulsively acting jet the flow gathers into one large eddy only under certain conditions, while for a continuously acting source it breaks into a system of eddies, which subsequently develop into the coherent vortex street similar to that formed behind a towed body. For large times, provided that the source Reynolds number is high enough, the vortex street becomes unstable and undergoes a transformation, during which its wavelength doubles. In some cases a tendency for the second similar transformation was evident, which signifies that the larger values of the source Reynolds number correspond to the large resulting eddies in a late wake. The similar tendency was observed for a stratified flow behind a sphere (Spedding et al, 1996).

This study was motivated by the idea, that there should be general similarity between stratified flows behind towed bluff bodies and moving jets. In both cases momentum is transported to the fluid, and it remains unchanged through the different stages of the flow development. It is known (at least for a homogeneous fluid), that at large distances where the flow in a turbulent co-flowing jet becomes self-similar, the laws, governing the jet spreading, are the same as those for an axially symmetric wake, i.e. $x^{-2/3}$ for a center-line velocity and $x^{1/3}$ for the width of the jet. The same laws were found to be valid in the late wake of a towed sphere in a stratified fluid (Spedding et al., 1996). Therefore, it could be conjectured that mostly momentum, and not the particular configuration of a solid boundary, determines the global characteristics of the far flow field. The presented arguments give reasoning for employing jets to study the far flow field in a stratified wakes. The variety of possible flow configurations with moving jets gives the opportunity to use this simple method in simulating much more complicated situations related, for example, to the late wakes signature behind maneuvering self-propelled bodies.

6. Acknowledgements

This study was supported by the Office of Naval Research and by a joint National Oceanic and Atmospheric Administration/Department of Defense Advanced Sensors Application Program.

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